DOCUMENT RESUME

ED 108 861 SE 030 720

AUTHOR Dunlap, Mike: And Others
TITLE Computers and Computer Literacy in the Algebra I and the Trigonometry Classroom. OCCE Special Report.
INSTITUTION Oregon Council for Computer Education, Portland.
PUB DATE Sep 74
NOTE 34p.: Contains occasional light and broken type.

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Algebra: *Computer Oriented Programs: Computer Programs: Computers: Curriculum Development: *Mathematical Applications: *Mathematics Curriculum: Mathematics Instruction: Secondary Education: *Secondary School Mathematics: *Trigonometry
IDENTIFIERS *Computer Literacy

ABSTRACT

The two sections found in this publication, "Integrating Computer Literacy With Existing Classes" and "Guide to Computer Augmented Trigonometry," were written by participants in two summer courses at the University of Oregon. The first section addresses the general topic of teaching computer literacy through existing classes, then it focuses on the specific course Algebra 1, including several specific examples of activities. The second section includes a variety of suggestions for using computers in trigonometry. While the main emphasis is on learning traditional course content, the student can also gain knowledge of the use of computers in mathematics in general. (MK)

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ED188361

SEPTEMBER 1974 SUGGESTED PRICE $1.50

OCCE Special Report

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A Publication of the Curriculum Group of the Oregon Council for Computer Education

COMPUTERS AND COMPUTER LITERACY IN THE ALGEBRA I AND THE TRIGONOMETRY CLASSROOM

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COMPUTERS AND COMPUTER LITERACY IN THE ALGEBRA I AND THE TRIGONOMETRY CLASSROOM

"Computer literacy" refers to the non-technical and low-technical aspects of the capabilities and limitations of computers, and of the social, vocational and educational implications of computers. There is general agreement among people knowledgeable in computer science that all students should acquire a reasonable level of computer literacy. This could be divided into two parts. First, there is the very general computer literacy that would be taught in a computer literacy course, or included in social studies, math, and science courses required of all students before high school graduation. One can think of this as providing the minimal skills and knowledge needed to cope with the computer aspects of life in our current society.

A second aspect of computer literacy can be associated with the various subject matter courses a student takes in high school. Thus, a student who studies higher level math courses should develop a math-oriented computer literacy compatible with his increased knowledge in mathematics; similar statements can be made about science courses, social science courses, etc. It is to this latter aspect of computer literacy that this document is directed.

The first section, "Integrating Computer Literacy with Existing Classes," is a paper prepared by nine teachers who were participants in the course, Teaching Computer Literacy, taught by David Moursund at the University of Oregon during Summer 1974. After addressing the general topic of teaching computer literacy through existing courses, it focuses upon the specific course, Algebra I, and provides a number of specific examples of things that could be done in that course.

The second section, "Guide to Computer Augmented Trigonometry," was written by two teachers who were participants in the course, Computer Curriculum Materials, taught by Mike Dunlap at the University of Oregon during Summer 1974. It contains a number of suggestions on using computers in a trigonometry course. The main emphasis in this case is the enhancement of learning of the traditional course content. However, a student in such a course will acquire an increased knowledge of the use of computers in mathematics.

INTEGRATING COMPUTER LITERACY WITH EXISTING CLASSES

by

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Why Should Computer Literacy be Taught at All?

Before we can consider how to teach Computer Literacy as part of existing high school courses, we must first decide whether computer literacy is important enough to be included within the high school curriculum at all.

New minimum state requirements for graduation from Oregon high schools state that students should be able to "understand basic scientific and technological processes", to function effectively "as a citizen in the community, state and nation, ..., as a citizen in interaction with his or her environment ... (and) ... as a consumer of goods and services." Consequently, any educational program which strives to prepare the student for his post-high school life is incomplete unless it introduces the student to the capabilities, applications and implications of computers.

At the present time, many of our students have some awareness of the use of computers in such fields as payroll, billing, credit and other financial transactions; automation in industry; transportation scheduling and reservation systems; the space program; and income tax processing. The use of computers in these and many other areas is continually increasing. Changes in the job market and in our patterns of daily life occur as computerization spreads to more and more fields.

It is important that today's youth also be aware of the implications of computer technology so that as adults they can be better prepared to adjust to life with the computer and to adapt the laws and practices of business, society and government to maximize the benefits and minimize the problems brought by extensive use of the computer. Some of our students will eventually have careers directly involved with computers; others will have non-computer careers in computer-oriented fields; and others will only have contact with computers in a passive way. However, even a person in the latter group (e.g. poets, artists, housewives, bartenders) will find a computer-based system to be the interface between himself and any large organization with which he deals (e.g. computerized mailings, reservations, bills, etc.). If he has no understanding of the system, he often feels overwhelmed, a powerless victim of automation.

Certainly the information a student learns about computers will vary from person to person based on individual interests and abilities. As stated in the goals for computer literacy drawn up by the OCCE, "All students should be computer literate at a level commensurate with their overall level of education."

What Should Be Included?

Computer literacy has been defined as "the non-technical study of the computer and its effect upon society."[1] More specifically, for a person to be computer literate, he should have some understanding or at least awareness in the following areas:

(1) A computer literate person understands the relationship between people and computers. He recognizes that the computer is a machine designed and built by people. He also understands that people determine the uses of computers and therefore are responsible for the actions of computers. He does not regard computers as substitutes for intelligence, but rather an extension of intelligence.

(2) A computer literate person is aware of the capabilities and limitations of computers. He has a knowledge of some of the applications of computers in business, industry, government, law enforcement, the military and education. He has some notion as to the methods used to apply the computer to various problems and the types of problems computers can and cannot solve. Thus he is more able to distinguish fact from nonsense and is less apt to be over-whelmed and intimidated by computers. He realizes that "The computer is incredibly fast, accurate and stupid. Man is unbelievably slow, inaccurate and brilliant. The marriage of the two is a force beyond calculation."[2]

(3) A computer literate person has some knowledge of the underlying structure of a computer, of computer hardware and software, and some of the standard vocabulary associated with the use of computers.

(4) A computer literate person is aware of some of the problems that exist or could exist in the future because of widespread use and misuse of computers. He is aware of the uses and implications of data banks; he has some idea of the effects of computers on industry, employment and government; and he has given some thought to the ethical considerations involved in widespread use of computers (invasion of privacy, loss of individuality, manipulation of people, etc.).

Where Should Computer Literacy Be Taught Within the School Curriculum?

The areas discussed above are ones which all students should be exposed to in a general course offered sometime during junior or senior high school. There is certainly more than enough content material for a semester course in computer literacy for the student who might have no subsequent exposure to computers in a more technical course such as math or science. Such courses exist in some schools already and hopefully will be added to the

[1] Mike Dunlap, "Computers in High School" as found in Proceedings of the Computers in Education Sections of Portland Meeting of NCTM, 1974.

[2] Leo Cherne quoted in Phi Delta Kappan, Oct. 1971, p. 122.

curriculum elsewhere as administration, teacher and/or student interest reaches an appropriate level.

Also, many high school students take science and math courses where they could acquire a more specialized technical knowledge about computers integrated with the subject matter. Likewise, some of the sociological aspects of computer use can and should be included in social studies classes and business uses of computers should be an integral part of the regular offerings of the business department.

It is the latter problem, increasing computer literacy within existing classes, that will be the subject of the remainder of this paper.

How Do We Teach Computer Literacy in Existing Classes?

There is the omnipresent problem of time. All courses have certain topics that must be included and teachers inevitably have trouble covering all of the subject material they would like to within the available class time. However, there are things that can be done which take very little or no extra class time than what is normally allotted to a given topic. (Specific examples related to Algebra I will be included in another section of this paper.) There are also topics that are less important now than in the pre-computer era (for example, in math, manual calculations of square roots and calculation with logarithms). Perhaps in the light of modern technology, teachers should consider a re-distribution of time allotments in order to allow introduction of more current topics.

If a teacher is aware of the importance of computers and is interested in conveying this feeling to his students, appropriate times to make incidental comments will come up naturally and will often spark students' interest. For example, a discussion on data banks or the role of computers in space efforts might be appropriate when some related development is in the news.

If a school has some computing facilities (as the majority of Oregon's high schools do), students can actually run their own programs, see relevant examples of programs the teacher has prepared, or run canned programs. In such situations, students will acquire some knowledge about types of problems that are appropriate for computer solution, some notion about their capabilities and some idea about computer hardware. However, even in situations where there are no computer facilities, there are still things that can be done in the field of computer literacy. And, of course, with electronic calculators so readily available now, there can be many new ways to vary class procedures and introduce students to some of the new technology. Several of these ways will be illustrated in the examples to follow.

Specific Examples of Teaching Computer Literacy

What follows is some examples of how topics from computer literacy could be incorporated into an existing algebra class. Although there are many areas from which examples could have been taken, we decided to choose from mathematics for three reasons:

(1) The area of expertise of the authors lies largely in mathematics.

(2) Math teachers are often more apt to have some knowledge and interest in computer science; and thus, can provide their students with more information and hopefully create more enthusiasm for further student learning.

(3) Mathematics and the study of computers form a two-way complementary relationship. The traditional problem solving approach to the study of mathematics can serve as model for the solution of computer problems and the computer can serve as a tool in the solution of mathematics problems, expecially when these problems are attacked using problem solving techniques. Thus, by combining math with computers, the student is better able to understand math's position in our modern technological society.

Example 1:

I. Area of Mathematics.
Flowcharting can be discussed relative to almost any algorithm or procedure in mathematics. This section will confine itself to the use of a flowchart in linear equation solving and later will list other areas that might easily follow this example.

II. How Flowcharting Relates to Computer Literacy.

A. Algorithms are an integral part of teaching an algebra course. These algorithms can easily be written in the form of a flowchart. Some teachers go so far as to list, step by step, procedures for many applications of concepts. This listing of a procedure is a flowchart.

B. One can, by paying careful attention to the total flowcharting concept, use it to provide the algebra student a bridge for the gap between actual problem solving and the human language definition of the problem.

C. The next natural step would be to extend the bridge at one end and make the problem-solver a computer. From this there are several possibilities for a semi-computer literate teacher to cause the literacy of his students to change.

1. An idea of the importance of a flowchart to problem solving with a computer.
2. The concept of the depth of understanding necessary for one to program a computer to solve a problem.

D. This allows us to discuss what types of problems might be good ones for using a computer. In a one-time only problem with relatively simple mathematics the job of telling the computer what to do would probably be harder than doing the problem by hand. A problem with complex mathematics or one to be done over and over might be best done using a computer.

1. The idea of what a computer can do. For example, it can follow a flowchart and solve a problem just like an algebra student, except a bit faster.

2. Flowcharting might bring us to a discussion of block type programming or structured programming.

III. Presentation of the Lesson.
The following flowchart (see Figure 1) can be followed to determine if an equation is linear (i.e., of the form ax+b=c) and if so, it's solution. The flowchart also will do more than one equation if supplied the values of the coefficients.

IV. Possible Student Activities.
Without facilities the teacher could simply discuss how the flowchart shows a way in which one can program a computer to do the problem. The computer's place can be taken by one of the students to simulate the solving. With facilities this can be extended to coding and running of the program.

V. Other Areas.
This study of flowcharting and computers can also be applied to many other areas, such as: signed number manipulation, factoring, and all types of equation solving.

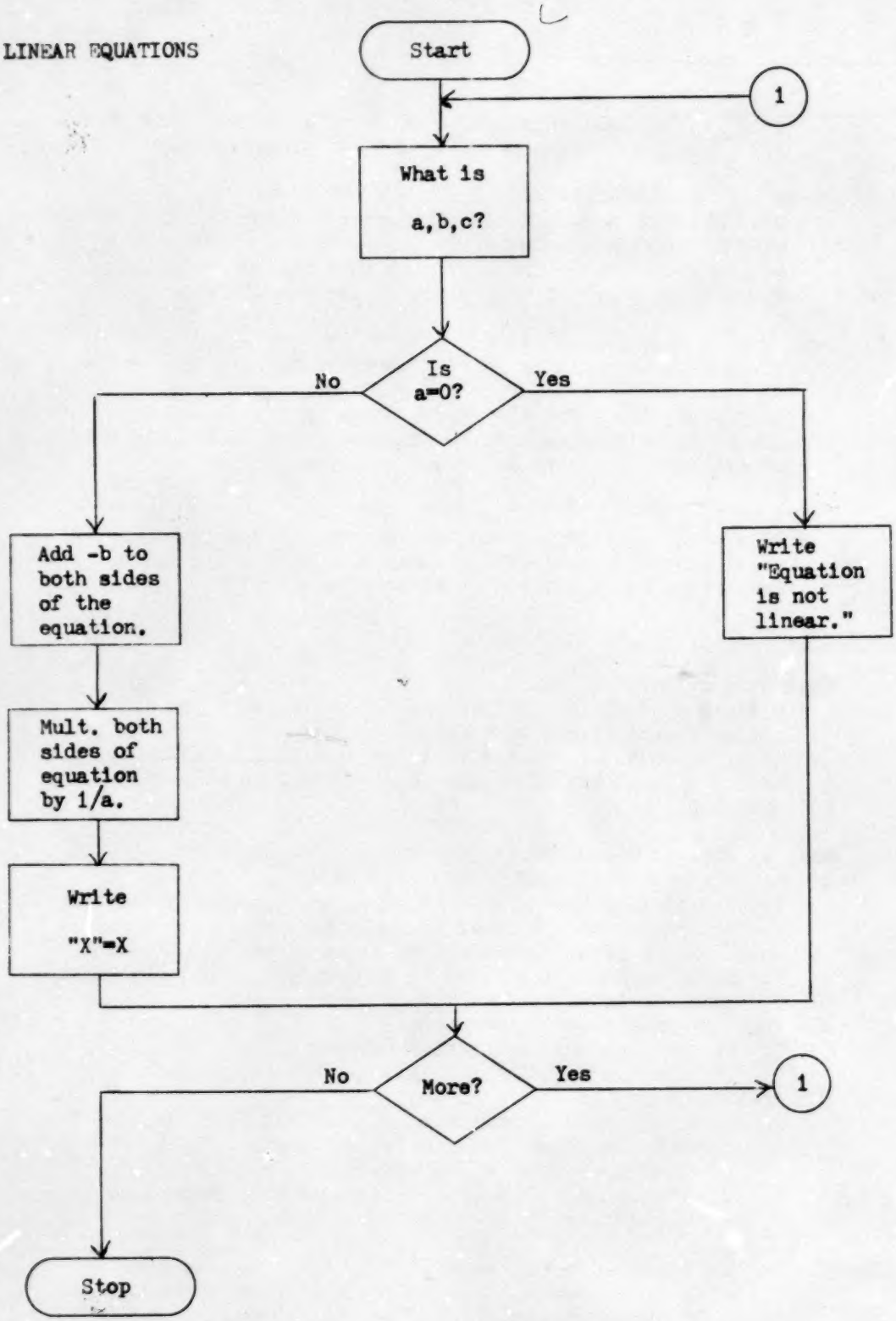
Example 2:

I. What Area of Math.
A standard topic covered in the early stages of any Algebra I course is operations with "signed" numbers. This is an example of how a computer print-out can be used to help students "discover" some of the rules for an operation, in this case subtraction.

II. What Aspects of Computer Literacy Will be Taught.
A. How do computers work?
1. The idea of a program that gives the computer directions.
2. An introduction to what is meant by a computer language.
3. The idea of a teletypewriter as an input/output device.
4. A notion about the speed of a computer (note .58 seconds printed at end of output).
B. What are some uses of computers?
1. To perform very rapid calculations.
2. To help teach students by giving them drill problems.
C. Additional Topics.
Hopefully this introduction will stimulate at least a few students to think about additional topics.
a. What does some of the code mean?
b. How could the program be changed to perform another operation?

III. How the Lesson Could be Presented.
The attached computer print-out could be thermofaxed onto a ditto master (or transparency) and distributed (or shown) to the class (see Figure 2). This could then be used to initiate a class discussion of the above computer-related topics. Then students could look at the output from the program and try to

LINEAR EQUATIONS



—Figure 1—

```
20  PRINT "X", "MINUS","Y","=","Z"
30  PRINT
40  PRINT
50  C=1
60  RANDOMIZE
70  X=INT(20*RND+(-10))
80  Y=INT(20*RND+(-10))
90  X=X-Y
100 PRINT X,"-",Y,"=",Z
110 PRINT
120 C=C+1
130 IF C>12 THEN 160
140 GO TO 70
150 PRINT
160 PRINT "NOW, CAN YOU FIGURE OUT THE RULE FOR SUBTRACTING SIGNED NUMBERS?"
170 PRINT "TRY THESE:"
180 PRINT
190 PRINT "-3-6=","5-(+7)=","-8-(-4)=","7-(-1)=","5-8="
200 END
```

X	MINUS	Y	=	Z
8	-	3	=	5
-3	-	0	=	-3
7	-	5	=	2
-1	-	1	=	-2
-7	-	-10	=	3
-3	-	-3	=	0
-1	-	9	=	-10
0	-	0	=	0
7	-	-4	=	11
-10	-	6	=	-16
7	-	-10	=	17
8	-	-7	=	15

```
NOW, CAN YOU FIGURE OUT THE RULE FOR SUBTRACTING SIGNED NUMBERS?
TRY THESE:

     -3-6=          5-(+7)=        -8-(-4)=       7-(-1)=        5-8=

TIME:  0.58 SECS.
```

(FIGURE 2)

figure out how the computer subtracted the signed numbers. After figuring out the process, students could try out the problems at the end and check results with the teacher.

IV. How Use of Terminal Facilities Could be Incorporated in the Lesson.
- A. The program could be modified into a CAI program where problems would be generated and students would input answers and then be told whether they were right or wrong.
- B. The program could be modified and run for other operations (addition, multiplication, division) by teacher or students.

Example 3:

I. What Area of Math.

One of the topics covered in an Algebra I class is solving simultaneous linear equations. This is an example of how a computer can tackle real life problems instead of artificial exercises.

II. What Aspects of Computer Literacy Will Be Taught.
- A. How could the computer be useful?
  1. The complexity of the problems does not affect the computer.
  2. Computers are much faster.
  3. Eliminates many student errors in calculations.
- B. Discuss how accuracy of computers will affect the answers.

III. How the Lesson Could be Presented.
- A. Start out with two linear equations with two unknowns of the general form:
  1. Ax+By=C (equation 1)
  2. Dx+Ey=F (equation 2)

  To solve this type of equation, you would do the following:

Multiply equation 1 by E.	A*E*x + B*E*y = C*E
Multiply equation 2 by -B.	-D*B*x + -B*E*y = -F*B
Add the two equations together.	A*E*x - D*B*x = C*E - F*B
Simplify.	(A*E - D*B)*x = C*E - F*B
Solve for x.	$x = \frac{C*E - F*B}{A*E - B*D}$
Similar process for y.	$y = \frac{C*D - F*A}{B*D - A*E}$

- B. Now to try some examples using this form:
  1. $2x + 3y = 7$ A=2 B=3 C=7
     $3x - 2y = 5$ D=3 E=(-2) F=5

$$x = \frac{(7*3) - (5*2)}{(-2*2) - (3*3)} = \frac{-14+(-15)}{-9+(-4)} = \frac{-29}{13} = \frac{29}{13}$$

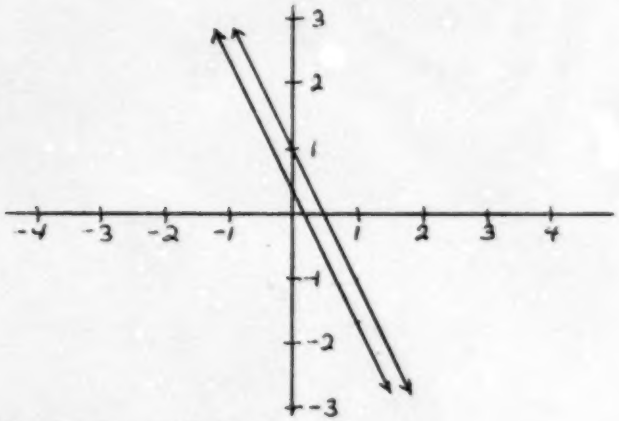
$$y = \frac{(7*3) - (5*2)}{(3*3) - (-2*2)} = \frac{21-10}{9+4} = \frac{11}{13}$$

2. $6x + 3y = 1$ A=6 B=3 C=1
   $4x + 2y = 2$ D=4 E=2 F=2

$$x = \frac{(1*2) - (2*3)}{(6*2) - (4*3)} = \frac{2-6}{12-12} = \frac{-4}{0}$$

Stop.

We now have to put in a check. What happens when (A*E) - (D*B) = 0? We can graph the equation and find out.



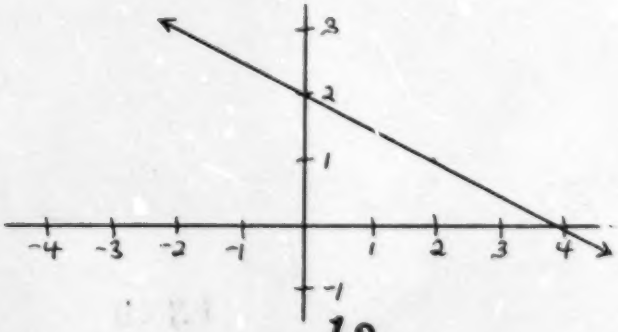
The lines are parallel.
Then our check would be:
If A*E - D*B = 0, then the lines are parallel.
*Note: B*D - E*A could also be used for the check.

3. $2x + 4y = 8$ A=2 B=4 C=8
   $3x + 6y = 12$ D=3 E=6 F=12

$$x = \frac{(8*6) - (4*12)}{(2*6) - (4*3)} = \frac{48 - 48}{12 - 12} = \frac{0}{0}$$

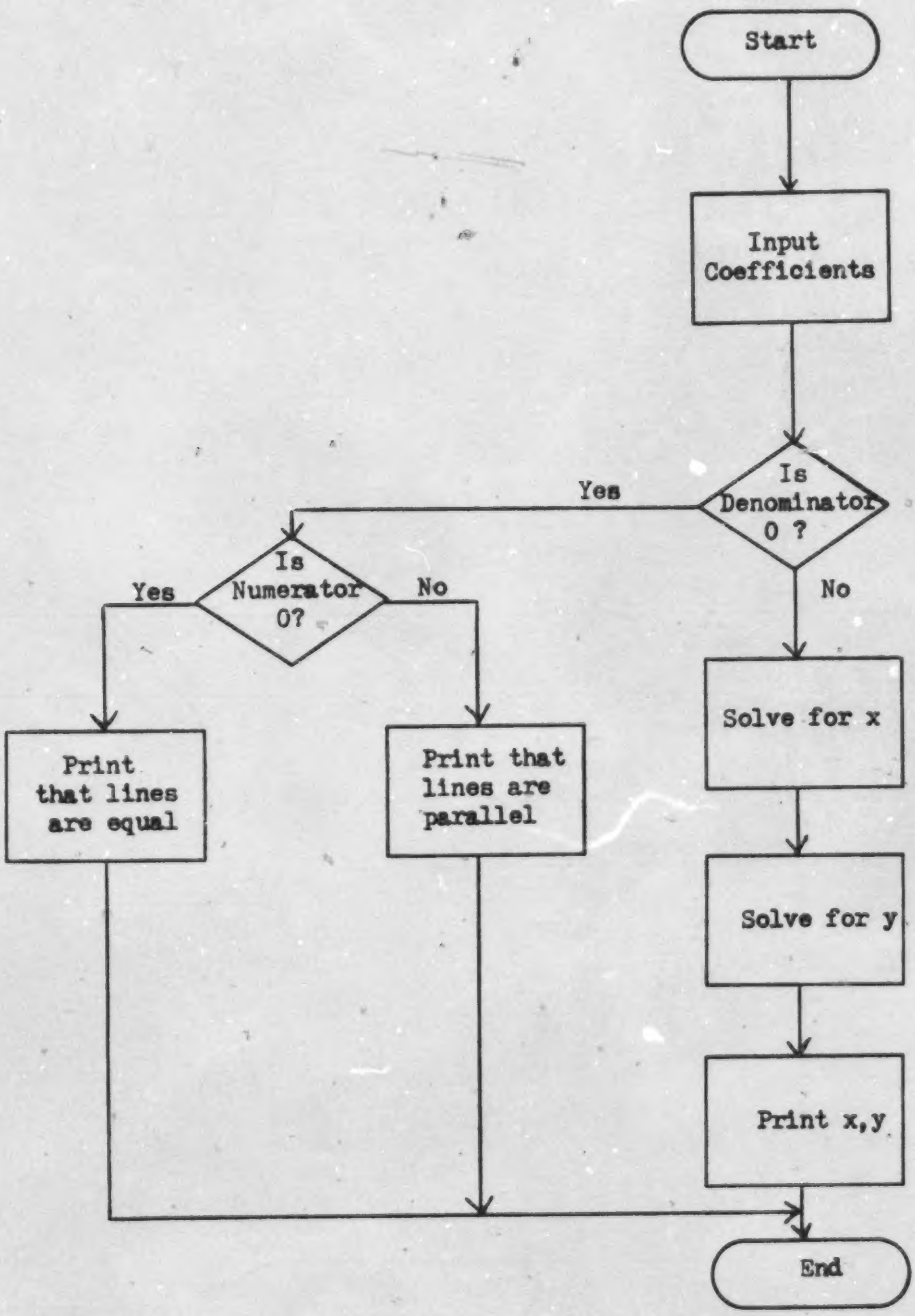
Stop.

Now let's graph this example:



So these equations are the same line.
Then our check would be:
If C*E - F*B = 0, then the equations are the same line.
*Note: C*D - F*A = 0 could also be used for the check.

A flowchart of this process is as follows:



IV. How use of Terminal Facilities Could be Incorporated in the Lesson.

A. The following program could be used to show the speed of the computer.

```
10  INPUT A,B,C,D,E,F
20  IF A*E - D*B = 0 THEN 70
30  LET X = (C*E - F*B)/(A*E - D*B)
40  LET Y = (C*D - F*A)/(B*D - E*A)
50  PRINT "X =" X, "Y =" Y
60  GO TO 110
70  IF C*E - F*B = 0 THEN 100
80  PRINT "THE TWO LINES ARE PARALLEL"
90  GO TO 110
100 PRINT "THE TWO LINES ARE EQUAL"
110 END
```

Now we could solve a system such as:

$$67.4x + 38.96y = 74.2$$
$$39.1x + 22.5y = 7.8$$

B. The program could also be modified for three equations with three unknowns or even larger.

Example 4:

I. What Area.

Most beginning algebra classes use square roots sometime during the year, and students almost invariably ask "What's a square root?" and how do they compute one. This example could be used to give them more insight into square roots and teach them some computer literacy at the same time.

II. Aspects of Computer Literacy to be Taught.

A. Idea of a program.
B. Concepts of speed and time saved using machine and implications of this.
C. Iterative processes (loops)
D. Limitations.
   1. Accuracy.
   2. Needs instructions.

III. How Lesson Might be Presented.

This example explores two methods of calculating square roots, the halving method and the quessing method. A teacher (using the flowcharts and guides on the following page) could present either one of these methods and have his students work with them and compare them. While going over the flowchart he could mention some of the various literacy aspects and discuss the implications.

IV. How the Use of Terminal Could be Incorporated.

If you have a terminal available it might be interesting to program it for the halving method and compare results for ac-

curacy and time with the computer's own subroutine for finding roots. The computer could also be programmed for the guessing technique to allow students to gain experience on the terminal. This would also allow them the opportunity of perhaps writing a program and seeing its implementation.

HALVING METHOD

Algorithm

1. Let n be the number we want to find the square root of.

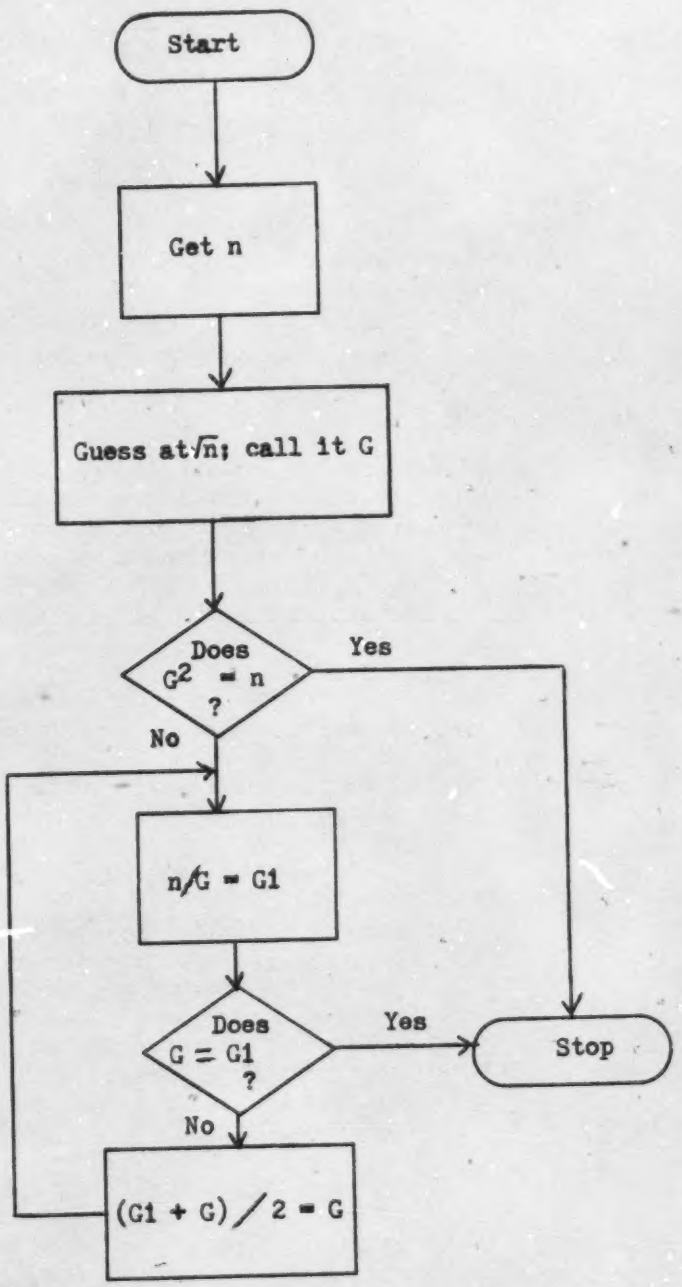
2. First we guess as close as possible. Call our guess G.

   If $G^2 = n$, then we are done; but if it does not, then:

3. Divide n by G and call our answer G1. (Note: when doing this by hand, the number of decimal places has no limit.) If G = G1 then we are done.

4. If G ≠ G1, then we must add G and G1 together and divide by 2. We call this our new G, and go back to (3).

Flow Chart



Example using a calculator

$\sqrt{7} = ?$

2	$2\sqrt{7} = 3.5$	$\frac{2}{3.5} \neq 1$	$\frac{2+3.5}{2} = 2.75$
2.75	$2.75\sqrt{7} = 2.5454545$	$\frac{2.5454545}{2.75} \neq 1$	$\frac{2.5454545+2.75}{2} = 2.6457752$
2.6457752	$2.6457752\sqrt{7}=2.6457506$*	$\frac{2.6457752}{2.6457506} \neq 1$	$\frac{2.6457752+2.6457506}{2} = 2.6457513$
2.6457513	$2.6457513\sqrt{7}=2.6457513$	$\frac{2.6457513}{2.6457513} = 1$*	

*Could stop here since $\frac{2.645}{2.645} = 1$ if we want only three places of accuracy.

*Must stop here since our calculator is only accurate for eight digits. Good place to do one by hand and see how accurate we can get. Could discuss what we loose in accuracy to gain speed. Is it signifigant?

NOTE: When doing this on a computer, you would want to check to see if $\frac{G}{G1}$ is in an interval of about one, since it is highly unlikely it will equal one because of round off. (i.e. If $.999 < \frac{G}{G1}$ and $\frac{G}{G1} < 1.001$, then we could say G is reasonably close to $\sqrt{n}$.

Example of guessing method

$\sqrt{7} = ?$ -- We know $2^2 = 4$ and $3^2 = 9$, so we guess:

2.5	$2.5^2 = 6.25$	too small
2.6	$2.6^2 = 6.76$	too small
2.7	$2.7^2 = 7.29$	too large
2.65	$2.65^2 = 7.0225$	too large
2.64	$2.64^2 = 6.9696$	too small
2.645	$2.645^2 = 6.996025$*	

*Could stop if 6.99 is close enough for our purposes, or you could continue to acquire a more accurate answer.

Example 5:

I. Area of Implementation.
Quadratic equations and the quadratic formula.

II. The Aspects of Computer Literacy Which Will be Taught.
A. Applications: real life calculations. Computers can be used as a desk calculator to do long, involved, repetitive calculations.
B. Problem solving: Algorithms and flow charts. The concept of an algorithm is fundamental to mathematics as well as to the field of computers.
C. Limitations: accuracy. A computer is limited to a certain number of significant figures. Gross errors can be introduced into certain kinds of calculations due to rounding.
D. Implications: mathematics education. The increased availability of computers and programable calculators that can solve complicated math problems which humans can solve by hand will definately have an effect on mathematics education.

III. Presentation of the Lesson
A. Discuss the algorithm for solving quadratic equations via quadratic formula and trace the flow chart given some values A,B, and C.
B. Real life problems.
1. Solution of real life problems in mathematics and science will most likely not involve "simple numbers" like the ones given in the text (A good example might be found in a chemistry text in the section on equilibrium.).
2. Try solving the following equation using the quadratic formula:
$$46.231X^2+.017X-2.008=0$$
3. How long would it take you to do this? How much more effort would it take than solving one of the problems in your text like:
$$2X^2+X-3=0$$
4. Discuss the use of computers and programable calculators in solving problems like these and how they reduce the need for hand calculations. Discuss the availability of programable calculators; such as the Hewlett-Packard 65. Should a human do what a machine can do with much greater speed and accuracy?
C. Discuss the kinds of problems that can arise when preforming operations like, $\sqrt{B^2-4AC}$ on a computer. Discuss the relationship of significant figures and round-off error to computer calculations.
D. Discuss the implications of computers in terms of mathematics education. What things should/will computers be doing for us in terms of mathematics?

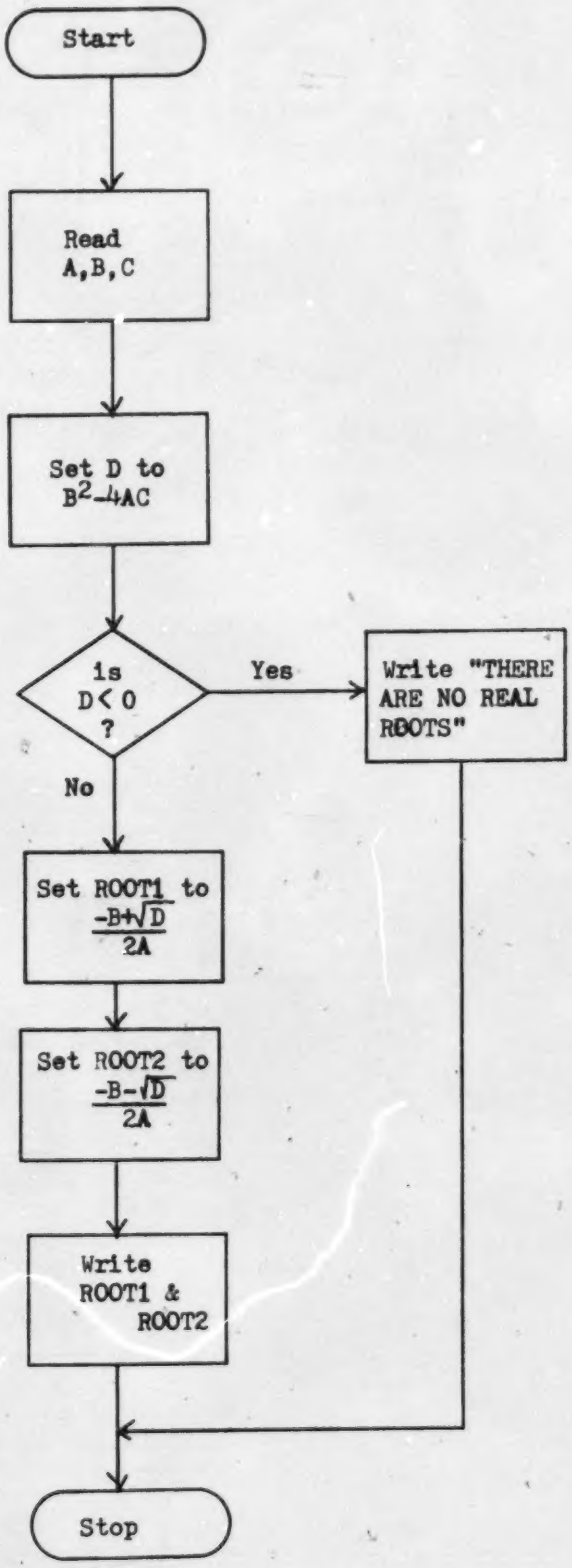
IV. Possible Student Activities.
A. Trace the flow chart given values for A,B, and C.
B. If familiar with flow charts, construct one from the algorithm and trace it.
C. If a terminal exists, write and run a program to solve any quadratic equation.

QUADRATIC FORMULA

Algorithm

1. Read A,B,C
2. Compute B^2-4AC and call it D.
3. If D<0, write "THERE ARE NO REAL ROOTS" and stop; otherwise, proceed to the next step.
4. Compute $\frac{-B+\sqrt{D}}{2A}$ and call it ROOT1.
5. Compute $\frac{-B-\sqrt{D}}{2A}$ and call it ROOT2.
6. Write the values of ROOT1 and ROOT2.
7. Stop.

Flow Chart



Topics For Further Consideration.

Many other topics exist in which ideas from computer literacy could be incorporated. Some examples of these are as follows:

I. From Mathematics.

A. Numerical Integration.
The value of π can be determined by throwing darts at a unit circle and finding the ratio of those that land in the circle to those that land inside a circumscribed square. A computer can simulate the same process by generating random numbers. This could be used to approximate the area under many curves.

B. Binary Vs. Decimal Number Systems.
Why does man use a decimal number system and the computer a binary number system? Why have the octal and hexadecimal number systems also become important? A study of these other systems may strengthen the understanding of the decimal system.

C. Graphing.
Using a computer to plot your students graphs takes less time, and allows the student to gain a better idea of what is happening. When he changes a variable or constant, he can immediately see the results.

D. Arithmetic Calculations.
With basic or general math students, electronic calculators can be used to eliminate the tedious work involved in such things as long division, thereby giving students more time to actually understand and solve real-life math problems without the frustration of complicated, usually meaningless calculations.

E. Trigonometry.
Calculators and/or computers can be used to allow students to work on more realistic trigonometry problems. The idea of a built-in function (e.g. sin, cos, log) can be mentioned as part of a discussion of computer capabilities.

F. Factoring Vs. Roots.
The computer can be used as a tool that uses binary searching techniques to find the roots and hense the factors of polynomial equations. Doesn't that make root searching techniques take on added importance? How about Newton's method and the calculator?

G. Associative and Commutative Properties.
A computer can easily test all the possibilities for a very large finite set, in comparison to the ability of a human, as long as the new operations can be described in terms of our basic operations. What implications does this ability produce for infinite sets and the computer?

H. Story Problems.
Much time and energy is spent trying to have students translate problems from English to Algebra, find a solution, and interpret the result. Should we still devote as much time to the topic? Who writes such programs?

I. Pythagorean Theorem.
A large portion of the difficulty here is the irrational nature of $c = \sqrt{a^2+b^2}$ in most cases. Should machine solutions (rational approximations) be stressed in lieu of unique real solutions? How often does one use $\sqrt{2}$ instead of 1.414 in real life? How about integral solutions of $c^2 = a^2+b^2$, $a \leq 100$, $b \leq 100$, $c \leq 100$?

J. Instructional Aids.
Suppose, in examining the behavior of sequences, students wish to know which of the following sequences grows the fastest: A binary sequence, sequence of squares, sequence of cubes, Fibonacci sequence, or sequence of factorials. Make a table printing out the first thirty of these sequences and discuss the results.

n	2^{n-1}	n^2	n^3	n!	Fibonacci
1	1	1	1	1	1
2	2	4	8	2	1
3	4	9	27	6	2
4	8	16	64	24	3
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

II. From Non-mathematics.

A. Home Economics.
It would be very handy to have a computer in the kitchen with which to store and retrieve recipies. It could also be used to make measurement conversions and to calculate partial recipies.

B. Psychology.
There are programs which play checkers, chess and converse with people. Do computers 'think' while they are doing these things? What is 'computer intelligence'? What is 'human intelligence'?

C. Art.
Programs exist which put out what is called computer 'art'. After seeing a demonstration of this via film or terminal one could ask, is this art? Whose? Machine or man's?

D. Social Studies.
At election time, a class could discuss the use of computers to predict winners with only very early returns available. The implications of data banks would be an important topic when discussing individual rights: how can the government balance efforts to achieve bureaucratic efficiency with the need to protect the liberties of individual citizens? How can we calculate the population of the world in the year 2000 if the annual increase is 2%?

E. Poetry.
Computers are able to 'write' poetry. How do you think human poets will accept their work? How would you convince someone that the computers work really is poetry?

F. Game Playing.
Since a computer can 'learn' to be a master at playing checkers, does this mean the computer might be able to learn to be a master diagnostician?

G. Science Experiments.
Some science experiments are too expensive and dangerous to do in the lab. Simulations are available for many of these experiments. This allows students the chance to do experiments he would not get to do otherwise, due to lack of time and money.

H. Statistics.
There are library programs available that will take any set of statistics and immediately give the standard deviation, mean, etc. They can be of great help to teachers and counselors.

Conclusion

All these examples are possibilities that were documented by a few people in a short period of time. Many more exist and even these need to be expanded and/or adapted to your particular classroom situation. But, it is a starting point on a trail that many believe we should be on--from textbook publishing companies--to leaders in computer education.*

It should be noted though, that when attempts are made to fit computer literacy into the high school curriculum, there is the need to consider the background of the teacher(s) who might be involved. As computer science is a relatively new field, most high school teachers have little or no formal training in the subject. Those with considerable background (if there are any within a particular school system) could play a leading role in the school's computer-related education and perhaps, hold workshops to introduce other teachers to the field. A teacher with only an introductory course (e.g. U. of O.'s 3 credit Concepts of Computing) and a real interest in the subject would certainly be in a position to introduce his students to the field of computer literacy. Even for a teacher with no background, who is interested in pursuing the subject, there are materials that are available to help him learn about this important field. A good introduction to the field would be a book such as The Computerized Society by Martin and Norman. Many other resources (including books, periodicals, and names of resource people) are described in the Computers in Education Resource Handbook. For any teacher to effectively teach computer literacy, the real key is teacher interest and enthusiasm which naturally spreads to students.

* See Algebra One from Silver Burdett Company, which includes a section on flowcharting and relevant problems that can be flowcharted and solved by computer at the end of each chapter, and "Computer Literacy" in the Computers in Education Resource Handbook by Dunlap and Moursund.

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GUIDE TO COMPUTER AUGMENTED TRIGONOMETRY

By Loran Sell and Tom Jepsen

The purpose of this material is to provide a possible skeleton outline and reference for the trigonometry teacher who would like to integrate some computer use into an existing trigonometry course. A minimal amount of the programming language BASIC is studied with the intent of strengthening understanding of and stimulating interest in the mathematical concepts. Programming is not absolutely necessary because it is possible to use the material contained here as a series of "canned" exercises.

The material is arranged under six broad headings or general topics of the typical trigonometry course: 1) Introduction, 2) trigonometric functions, 3) Identities, 4) Amplitude, period, and phase shift, 5) Right triangles, 6) Solving triangles using the law of sines and law of cosines. Each general topic is presented in an outline format using: mathematical concepts, new computer topics, suggested activities, a sample activity (presented in some detail), and sources of information.

It has been assumed the teacher has a working knowledge of BASIC and access to a computer or terminal. No attempt has been made to suggest methods of teaching, but to suggest a list of activities that might be used to take advantage of the computer in a trigonometry course.

I. Introduction to Computer Augmented Trigonometry
   A. Mathematical Concepts
      Most trigonometry texts start with a review of math topics, such as the distance function, coordinate system, definition of functions, arc distance, and other review topics. This mathematical review provides an opportune time for an introduction to the computer.
   B. New Computer Topics
      1. The first computer topic should be log-on and log-off procedure for the system to be used.
      2. An explanation and demonstration of running "canned programs"
      3. An explanation of the basic parts of a computer, aids in student understanding.
         a. Central Processing Unit
         b. Input and Output Units
         c. Memory Units
      4. The introduction for the computer language BASIC should include the following:
         a. The PRINT statement with the use of commas for spacing output.
         b. The LET statement for initializing, incrementing, and assignment statements.
         c. The END statement
   C. Suggested Activities
      1. The teacher or an experienced student should demonstrate

a run of a "canned program" such as those listed below.

a. Distance.
b. Program to compute the perimeter of a polygon.
c. Program to test triangles for scalene, isosoles, or equilateral.
d. Program to print out the square and cube of a number.

2. The students may be one or more "canned programs" to run either individually or in small groups.
3. Play a computer game using a group of students to play the part of each computer unit. Example: Each student in the memory unit could be a storage location used in running a simple program.
4. Assign the students a program to write that will change 37° centigrade to fahrenheit.

D. Sample Activity

1. Since this is probably the first encounter with a computer for many of the students, divide the class into groups of three to five students. If possible, include in each group a student with some computer experience. Assign the program "DISTANCE" to each group and allow class time for their run. Additional runs by individuals might take place during free time.
2. Listing of "DISTANCE"

```
10 REM PROGRAM TO FIND THE DISTANCE BETWEEN ANY 2 POINTS ON THE
20 REM COORDINATE PLANE.
30 PRINT "DO YOU WANT INSTRUCTIONS? TYPE 0 FOR NO OR 1 FOR YES"
40 INPUT I
50 IF I=0 THEN 170
60 REM
70 REM**************INSTRUCTIONS*****************************
80 REM
90 PRINT "THIS IS A PROGRAM THAT USES THE DISTANCE FORMULA"
100 PRINT"D=SQR((X(1)-X(2)) 2+(Y(1)-Y(2))↑2) TO DETERMINE"
110 PRINT"THE LINEAR DISTANCE BETWEEN ANY 2 POINTS ON THE
    COORDINATE"
120 PRINT"PLANE. WHEN ASKED FOR A POINT, TYPE THE X VALUE, Y
    VALUE"
130 PRINT "THEN A CARRIAGE RETURN.  EXAMPLE:  TO ENTER THE
    POINT"
140 PRINT"(3,5) TYPE 3,5 CR"
150 PRINT
155 REM
160 REM **************OBTAINING USER INPUT********************
165 REM
170 PRINT"POINT NUMBER ONE IS";
180 INPUT X(1),Y(1)
190 PRINT "POINT NUMBER TWO IS";
200 INPUT X(2),Y(2)
210 REM
220 REM **************CALCULATIONS AND OUTPUT*****************
```

```
230 REM
240 LET D=SQR((X(1)-X(x))↑2+(Y(1)-Y(2))↑2)
250 PRINT"THE DISTANCE BETWEEN (";X(1);Y(1);")AND"
255 PRINT"(";X(2);") IS ";D
260 PRINT "WOULD YOU LIKE TO TRY ANOTHER TWO POINTS?"
265 PRINT "TYPE 0 FOR NO AND 1 FOR YES";
270 INPUT R
280 IF R=1 THEN 170
290 PRINT "I HOPE THAT YOU HAVE ENJOYED YOUR ENCOUNTER WITH"
300 PRINT" THE COMPUTER.  I HAVE CERTAINLY ENJOYED WORKING FOR
    YOU."
310 PRINT" COME BACK AND SEE ME AGAIN, O.K.?  GOOD-BYE"
320 END
```

3. Sample run of "DISTANCE"

```
DO YOU WANT INSTRUCTIONS? TYPE 0 FOR NO OR 1 FOR YES
?1
THIS IS A PROGRAM THAT USES THE DISTANCE FORMULA
D=SQR((X(1)-X(2))↑2+(Y(1)-Y(2))↑2) TO DETERMINE
THE LINEAR DISTANCE BETWEEN ANY 2 POINTS ON THE COORDINATE
PLANE.  WHEN ASKED FOR A POINT, TYPE THE X VALUE, Y VALUE
THEN A CARRIAGE RETURN.  EXAMPLE:  TO ENTER THE POINT (3,5)
TYPE 3,5 CR

POINT NUMBER ONE IS ?4,8
POINT NUMBER TWO IS ?5,9
THE DISTANCE BETWEEN ( 4 8 ) AND
( 5 9 ) IS 1.41421
WOULD YOU LIKE TO TRY ANOTHER TWO POINTS?
TYPE 0 FOR NO AND 1 FOR YES ? 1
POINT NUMBER ONE IS ?12,10
POINT NUMBER TWO IS ? 35,72
THE DISTANCE BETWEEN ( 12 10 ) AND
(  35  72  ) IS 66.1287
WOULD YOU LIKE TO TRY ANOTHER TWO POINTS?
TYPE 0 FOR NO AND 1 FOR YES ?0
I HOPE THAT YOU HAVE ENJOYED YOUR ENCOUNTER WITH THE
COMPUTER.  I HAVE CERTAINLY ENJOYED WORKING FOR YOU.
COME BACK AND SEE ME AGAIN, O.K.?  GOOD-BYE
```

E. Sources

1. Basic Programming, second edition, by John G. Kemeny and Thomas E. Kurtz, Wiley 1971
2. Trigonometry, second edition, by Eldridge P. Vance, Addison-Wesley 1969
3. Second Course In Algebra And Trigonometry With Computer Programming, a publication of the Colorado Schools Computing Science Curriculum Development Project, directed by Chuan C. Feng, 1969
4. REACT Program Library, Northwest Regional Educational Laboratories, Portland, Oregon

II. TRIGONOMETRIC FUNCTIONS

A. Most trigonometry texts contain a section that is devoted to introducing the trig. functions and the fundamental identities ($\frac{\sin X}{\cos X} = \tan X$, $\sin^2 X + \cos^2 = 1$, etc.). This section describes activities using the computer that might be appropriate to use while studying such material.

B. NEW COMPUTER TOPICS

1. STATEMENTS
   READ
   DATA
   GO TO
2. flow charting
3. use of system functions (SIN,COS,TAN)

C. SUGGESTED ACTIVITIES

1. Use the SIN, COS, TAN functions in BASIC to evaluate the sine, cosine, or tangent of specific numbers.

```
EXAMPLE PROGRAM:  120  LET D=60
                  130  LET R=3.1415960*D/180
                  140  PRINT SIN(R),COS(R)
                  150  END
                 RUNNH
                 0.866025        0.500001
```

NOTE: 130 changes units from degrees to radians
It might be appropriate here to discuss round-off when using the computer

2. A discussion of how the computer evaluates the trigonometric functions (power series approximations; might take place. It can be pointed out that the values arrived at are approximations (precise to about 6 digits). Some students might wish to write their own programs to approximate the trigonometic functions using:

$$\sin(X) = X - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \cdots$$

$$\cos(X) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^{n+1} \frac{x^{2n-2}}{(2n-2)!} + \cdots$$

where X is in radians.

3. Use the computer to plot graphs of the trig. functions. A number of programs are available for this purpose.
   Hewlett Packard - PLOT
   REACT - PLOT
   paper tapes available

D. SAMPLE ACTIVITY:

Generating tables for graphing of TRIG. FUNCTIONS.

The teacher might start with an activity such as #3 above and discuss the inefficiency of the program for generating values for graphing a function. This might lead to a class

development of a flow chart like the one following and a program similar to one of the two given. There are two ways the activity might be developed: 1) the program might be introduced and the flow chart written during class discussion in order to determine what the program does, 2) present the flow chart and use it as a lead in to developing a program. Students then could alter the program slightly to generate different tables.

PROGRAM 1.

```
 5 PRINT "ANGLE", "RADIANS", "SIN", "COS" (optional if print-
                          ing of alphabetic data has been discussed)
10 READ X
15 DATA 30,60,90,80,45
20 LET R=3.14159*X/180
25 PRINT X, R, SIN(R), COS(R)
30 GO TO 10
35 END

RUNNH
ANGLE  RADIANS    SIN  COS
  30   0.523598  0.5  0.866026
  60   1.0472
  90
  80         ETC.
  45
   ? OUT OF DATA IN LINE 5
```

PROGRAM 2. (using a FOR-NEXT loop)

Note: the material in this program could be introduced later in the unit of amplitude, period, and phase angles of trig. functions.

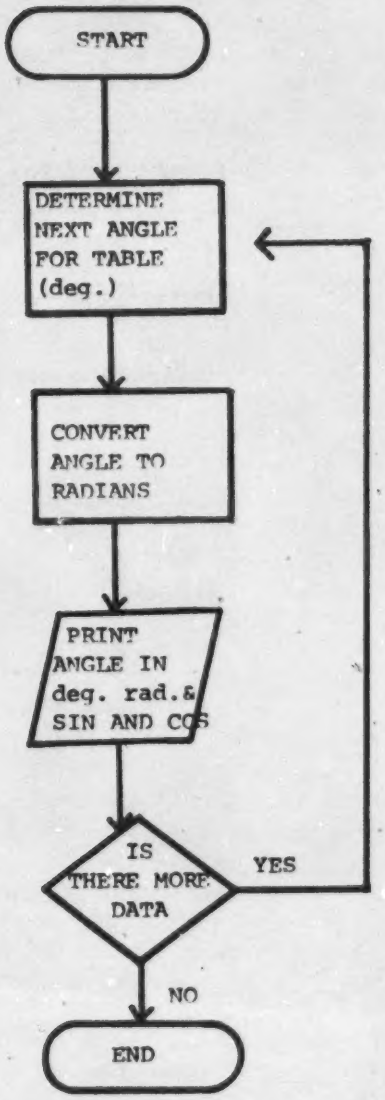
```
5  READ B,C
6  DATA -12,12
10 LET P1=3.141592653
20 PRINT "X", "X IN RAD.", "SIN(X)", "COS(X)"
30 FOR A=B TO C
40 LET X=30*A
50 LET R=x*P1/180
60 PRINT X, R, SIN(R),COS(R)
70 NEXT A
80 END
RUN NH
```

X	RADIANS	SIN(X)	COS(X)
-360	-6.28319	1.87254E-7	1
-330	-5.75959	0.5	0.866025
-300	-5.23599	0.866025	0.5
-270	-4.71239	1	-2.34067E-8
-240	-4.18879	0.866025	-0.5
-210	-3.66519	0.5	-0.866025
-180	-3.14159	-9.36268E-8	-1
-150	-2.61799	-0.5	-0.866025
-120	-2.0944	-0.866025	-0.5
-90	-1.5708	-1	4.47035E-8
-60	-1.0472	-0.866025	0.5
-30	-0.523599	-0.5	0.866025

(etc. up to 360 degrees)

NOTE: We are being rather inefficient here by not using the STEP notation, and by not combining some statements. The thought is each statement should deal with only one step of the problem if possible.

FLOW CHART 1.



E. SOURCES

1) Feng Chuan C., SECOND COURSE IN ALGEBRA AND TROGONOMETRY WITH COMPUTER PROGRAMMING, The Colorado Schools in Computing Science Curriculum Development Project, 1973. (chapter 6)
2) Vogeli, Bruce; Moredock, H. Stewart; Prevost, Fernand; Torell, Arthur C., ALGEBRA II AND TRIGONOMETRY, Silver Burdett, 1972. (general)
3) PLOT - a program to plot a given function on the terminal, is listed in the TIME-SHARED BASIC PROGRAM LIBRARY HANDBOOK;

Hewlett-Packard, Software Center, 11000 Wolfe Road, Cupertino, Cal.

III. MULTIPLE ANGLE FORMULAS AND IDENTITIES

A. The math topics that might be studied in this section would include the sum and difference formulas, double angle formulas, and general trig. identities. The computer can be used to develop an inductive approach to the concepts.

B. NEW COMPUTER TOPICS

1. Printing of alphabetic material (for table and column headings)

C. SAMPLE ACTIVITY

The members in the class are given "possible identities" to investigate using the computer and hypothesize, based on output, as to the truth or falsity of the equation.
EXAMPLE: Is the following equation true for all angles X for which the equation makes sense?

$$\sin(3X)=3\sin(X)-4\sin^3(X)$$

The class might work in small groups, write programs to test the equation for various angles, and form the hypothesis as to its validity.

```
5  READ X
6  DATA 17,30,150,235,341
10 LET R=3.14159*X/180
20 PRINT "FOR R=", R
30 PRINT "SIN(3*R)=", SIN(3*R)
40 PRINT "3*SIN(R)-4*SIN(R)↑3=", 3*SIN(R)-4*SIN(R)↑3
50 PRINT
60 GO TO 5
70 END
RUN
```

Topics that might be discussed as a class prior to or after the activity might include:

1. How many angles should be tested? How should they be chosen? If an equation holds for 5 angles is it reasonable to hypothesize that it's true for all angles? Would we want to test an equation with the angles 0, 180, 360 degrees?
2. Will the fact that the computer only approximates the values effect the results? Should we expect any discrepencies in true identities?

D. SOURCES

1) Colorado Project, Feng, Chuan C., SECOND COURSE IN ALGEBRA AND TRIGONOMETRY WITH COMPUTER PROGRAMMING, The Colorado Schools Computing Science Curriculum Development Project, 1973, (chapter 6)

IV. PERIOD, PHASE SHIFT, AND AMPLITUDE OF CIRCULAR FUNCTIONS:

A. General study of the equation y=a*f(b(x-c))+d where a,b,c,d are real numbers and f is one of the six trig. functions.

ex.) y=5sin x

y=3 cos 2(x+π/4)

B. NEW COMPUTER TOPICS:

1. FOR-NEXT LOOPS

C. SUGGESTED ACTIVITIES

1. Use one of the canned programs to plot a graph of a specific equation. The class might discuss the equation, hypothesize as to the shape of its graph and run the program PLOT to test the hypothesis for immediate reinforcement.
2. Use a canned program to produce tables of values for the functions under consideration. (see GENFUN in sources)

D. SAMPLE ACTIVITY:

1. As a class activity the teacher might introduce a program similar to the one below to generate a table of values for a given equation. Small groups might then write similar programs for different equations to test the effect of varying the values of one of the constants in the general equation given above.

ex.) y=cos x

y=4cos x

y=1/4cos x

```
5   READ B,C
6   DATA -12, 12
10  LET P1=3.141592653
20  PRINT "ANGLE", "RADIANS", "4*COS(R)","COS(R)", "14/*COS(R)"
30  FOR A=B TO C
40  LET X+30*A
50  LET R=X*P1/180
60  PRINT X, R, 4*COS(R), COS(R), 1/4*COS(R)
70  NEXT A
80  END
RUNNH
```

ANGLE	RADIANS	4COS(R)	COS(R)	1/4COS(R)
-360	-6.28319	4	1	0.25
-330	-5.75959	3.4641	0.866025	0.216506
-300	-5.23599	2.	0.5	0.125
-270	-4.71239	-9.36268E-8	-2,34067E-8	-5.85167E-9
-240	-4.18879	-2.	-0.5	-0.125
-210	-3.66519	-3.4641	-0.866025	-0.216506
-180	-3.14159	-4	-1	-0.25
-150	-2.61799	-3.4641	-0.↑C	
-2.0944	↑C			

(etc. up to 360 degrees)

E. SOURCES

1. GEFUN - a program to generate tables of ordered pairs for any function specified by the user, REACT Program Listings, Northwest Regional Educational Laboratories.

V. Solving Right Triangles

A. Mathematical Concepts

Most circular functions texts approach the solving of right triangles as an extension of circular functions. Thus, sine-side opposite divided by hypotenuse and cosine-side adjacent divided by hypotenuse, etc., should be defined and developed in this section. The computer should be used in this section as a tool to aid in the calculations encountered when using these definitions to solve right triangles.

B. New Computer Topics

1. Problem layout, using flow charts to accurately define the problem, should be discussed.
2. The REM statement is to be used for documentation.
3. Discussion of the INPUT statement should include the printing of an initial question.

C. Suggested Activities

1. Write a program to solve for an unknown side given the hypotenuse and an acute angle.
2. Write a program to solve for an unknown angle given the side opposite and the side adjacent.
3. Make a flow chart outlining the process required to solve for the missing parts of any right triangle given two defining parts.
4. Demonstrate a program to solve any right triangle.
5. As a class project, write a program that will:
   a. Solve any right triangle
   b. Solve a "sliding ladder" problem
   c. Break a vector into its components
   d. Determine the area of a right triangle given the hypotenuse and one acute angle.

D. Sample Activity

1. Objectives
   a. The student will demonstrate an ability to use the statements REM and INPUT in a program
   b. The student will demonstrate an understanding of solving a right triangle by writing a program to solve the triangle, given the hypotenuse and one acute angle
   c. The student will demonstrate an understanding of the SIN function by using it in his program.
2. Assignment

   Write a program to solve any right triangle for all sides and angles if one acute angle and the hypotenuse are given. The program should be interactive, asking for the user to input the necessary information. Make use of the REM statement to document your program.
3. Sample Output

```
WHAT IS THE LENGTH OF THE HYPOTENUSE? 2
WHAT IS THE MEASURE OF THE KNOWN ANGLE? 30
SIDE 1 HAS LENGTH 1
SIDE 2 HAS LENGTH 1.71
ANGLE 2 HAS MEASURE 60
```

E. Sources
1. Second Course In Albegra And Trigonometry With Computer Programming, a publication of the Colorado Schools Computing Science Curriculum Development Project, Director Chuan C. Feng, 1969
2. Basic Programming, second edition, by John G. Kemeny and Thomas E. Kurtz, Wiley 1971
3. Trigonometry, second edition, by Eldridge P. Vance, Addison-Wesley 1969

VI. Solving Triangles using the Law of Sines and Law of Cosines

A. Mathematical Concepts

Before attempting the use of the computer to solve triangles, the student should be familiar with the concept that given three parts of a triangle, the triangle may or may not be defined, and if so, outline a process using the law of sines and/or the law of cosines to solve for the three unknown parts.

B. New Computer Topics
1. The IF-THEN statement should be defined for use with equalities and inequalities.
2. The GOSUB may be omitted if time does not permit its development.
3. The defining of functions may be introduced to define ARCOS Z = ARCTAN(SQR(1-(COS(Z)↑2)/COS(Z)).

C. Suggested Activities
1. Write a program to solve an oblique triangle in one of the following cases:
   a. Given two sides and the included angle
   b. Given two angles and the included side
   c. Given two angles and the side opposite one
   d. Given the length of each side
2. Run one or more of the following "canned programs."
   a. A program to solve a triangle given two sides and the angle opposite one
   b. REACT library program "COSSIN"
   c. Basic Programming "TRIG 1", "TRIG 2", "TRIG 3"
   d. Modern Trigonometry "Computer Investigations" pages 181-183
3. Make a flow chart for the solving of any triangle given three of the six parts.
4. As a class project, write a program to solve any triangle given any three of the six parts.

D. Sample Activity
1. The activity is the writing of a program to solve any triangle given any three of the six parts. This activity should be preceded by a discussion of what defines a triangle.
2. The following assignments could then be made, either to individuals or to groups.
   a. Make a flow chart of the entire process, without being concerned with how each case is actually solved

b. Write a program to solve the triangle given two sides and the included angle
c. Write a program to solve the triangle given two angles and the included side
d. Write a program to solve the triangle given two angles and the side opposite one
e. Write a program to solve the triangle given all three sides
f. Write a program to solve the triangle given two sides and the angle opposite one (this may be divided into two parts)
g. Write a program to test for and print an error message if a triangle is not defined

3. When each of the above assignments are completed, then the class, as a whole, should put the parts together into one program. The class could be led into student discovery of some of the following observations, which when considered may make a neater and shorter program.
   a. The user may interact with the program by inputting a number which corresponds to the case (A.S.A., S.A.S., etc.), and the data
   b. Several sequences of operations are repeated in more than one case, they may be appropriate for a subroutine
   c. Assignment "c" above could be considered as a special case of "d"
   d. To run the program several times in a row, an input of "1" meaning go to the beginning or "0" meaning end could be included
4. When the entire program has been completed the teacher or one of the students should program the computer, then several test runs should be made by the class.
5. Some discussion might take place as to how this program could be modified, to be useful to other users as a "canned program".

E. Sources

1. Trigonometry, second edition, by Eldridge P. Vance, Addison-Wesley, 1969
2. Basic Programming, second edition, by John G. Kemeny and Thomas E. Kurtz, Wiley, 1971
3. REACT Program Library
4. Second Course in Algebra and Trigonometry With Computer Programming, a publication of Colorado Schools Computing Science Curriculum Development Project, directed by Chuan C. Feng, 1969
5. Modern Trigonometry, by Wootan, Beckenback, Buchanan, and Dolciani, Houghton Mifflin, 1973